



**SPRING GROVE AREA SCHOOL DISTRICT**



**PLANNED COURSE OVERVIEW**

<b>Course Title:</b> Advanced Placement Calculus BC <b>Grade Level(s):</b> 12 <b>Units of Credit:</b> 1 <b>Classification:</b> Elective	<b>Length of Course:</b> 30 cycles <b>Periods Per Cycle:</b> 6 <b>Length of Period:</b> 43 minutes <b>Total Instructional Time:</b> approx. 130 hours
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***Course Description***

This is an advanced course designed to prepare students for the Advanced Placement (AP) Calculus BC College Board examination. The course goals in compliance with College Board include the following: Working with functions represented in a variety of ways: graphical, numerical, analytical, or verbal; understanding the meaning of the derivative in terms of rate of change and local linear approximation and using derivatives to solve a variety of problems; understanding the meaning of the definite integral both as a limit of Riemann sums and as the net accumulation of change and using integrals to solve a variety of problems; understanding the relationship between the derivative and the definite integral as expressed in both parts of the Fundamental Theorem of Calculus; working with parametric, polar, and vector functions; understanding polynomial approximations and series; communicating mathematics both orally and in well-written sentences and learning how to explain solutions to problems; modeling a written description of physical situation with a function, a differential equation, or an integral; using technology to help solve problems, experiment, interpret results, and verify conclusions; determining the reasonableness of solutions, including sign, size, relative accuracy, and units of measurement; and ultimately developing an appreciation of calculus as a coherent body of knowledge and as a human accomplishment.

***Instructional Strategies, Learning Practices, Activities, and Experiences***

Anticipatory Sets	Flexible Groups	Projects
Assessments	Graphic Organizers	Teacher Demonstrations
Bell Ringers	Guided Practice	Technology Integration
Class Discussions	High-Level Questioning	Videos/DVD's
Closure	Homework	Wait Time
Critical Thinking	Posted Objectives	

***Assessments***

Assessments (Teacher-Created, College Board) Higher-Level Questioning	Projects	Classwork
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## ***Materials/Resources***

Calculus of a Single Variable: Eighth Edition  
(Larson, Hostetler, Edwards)

Internet Resources

College Board Materials

**Adopted:** 5/20/13

**Revised:** 5/20/2019

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<b>I. Functions, Graphs, and Limits</b>	
<b>The Standards of Mathematical Practices</b>	
<p><b>Make sense of problems and persevere in solving them.</b>  <b>Construct viable arguments and critique the reasoning of others.</b>  <b>Use appropriate tools strategically.</b>  <b>Look for and make use of structure.</b></p>	<p><b>Reason abstractly and quantitatively.</b>  <b>Model with mathematics.</b>  <b>Attend to precision.</b>  <b>Look for and express regularity in repeated reasoning.</b></p>
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(A) Analysis of Graphs	1. Use analytic information to predict and explain graphical behaviors.
(B) Limits of Functions (Including one-sided Limits)	<ol style="list-style-type: none"> <li>1. Develop an intuitive understanding of the limiting process.</li> <li>2. Calculate limits using Algebra.</li> <li>3. Estimate limits from graphs, tables, and data.</li> </ol>
(C) Asymptotic and Unbounded Behavior	<ol style="list-style-type: none"> <li>1. Describe asymptotes in terms of graphical behavior.</li> <li>2. Describe asymptotic behavior in terms of limits involving infinity.</li> <li>3. Compare relative magnitudes of functions and their rates of change (for example, contrasting exponential growth, polynomial growth, and logarithmic growth).</li> </ol>
(D) Continuity as a Property of Functions	<ol style="list-style-type: none"> <li>1. Develop an intuitive understanding of continuity.</li> <li>2. Describe continuity in terms of limits.</li> <li>3. Demonstrate knowledge of the Extreme Value Theorem and the Intermediate Value Theorem.</li> </ol>
(E) Parametric, Polar, and Vector Functions	1. Demonstrate knowledge of parametric, polar, and vector functions.

II. Derivatives	
CONTENT/KEY CONCEPTS	OBJECTIVES/STANDARDS
(A) Concept of the Derivative	<ol style="list-style-type: none"> <li>1. Represent the derivative graphically, numerically, and analytically.</li> <li>2. Interpret the derivative as an instantaneous rate of change.</li> <li>3. Use the limit process to define the derivative.</li> <li>4. Explain the relationship between differentiability and continuity.</li> </ol>
(B) Derivative at a Point	<ol style="list-style-type: none"> <li>1. Find the slope of a curve at a point (with emphasis on points where there are vertical tangents and points which there are no tangents).</li> <li>2. Determine the tangent line to a curve at a point.</li> <li>3. Calculate local linear approximation.</li> <li>4. Calculate instantaneous rate of change as the limit of average rate of change.</li> <li>5. Approximate the rate of change from graphs, and tables of values.</li> </ol>
(C) Derivative as a Function	<ol style="list-style-type: none"> <li>1. Describe corresponding characteristics of graphs of <math>f</math> and <math>f'</math>.</li> <li>2. Explain the relationship between the increasing and decreasing behavior of <math>f</math> and the sign of <math>f'</math>.</li> <li>3. Utilize the Mean Value Theorem and its geometric interpretation.</li> <li>4. Translate verbal descriptions into equations involving derivatives and vice versa.</li> </ol>

II. Derivatives (continued)	
CONTENT/KEY CONCEPTS	OBJECTIVES/STANDARDS
(D) Second Derivative	<ol style="list-style-type: none"> <li>1. Explain the corresponding characteristics of the graphs of <math>f</math>, <math>f'</math>, and <math>f''</math>.</li> <li>2. Describe the relationship between the concavity of <math>f</math> and the sign of <math>f''</math>.</li> <li>3. Determine points of inflection and state the places where concavity changes.</li> </ol>
(E) Applications of Derivatives	<ol style="list-style-type: none"> <li>1. Analyze curves, including the notions of monotonicity and concavity.</li> <li>2. Analyze planar curves given in parametric form, polar form, and vector form, including velocity and acceleration.</li> <li>3. Solve optimization problems, both absolute (global) and relative (local) extrema.</li> <li>4. Model rates of change, including related rates.</li> <li>5. Use implicit differentiation to find the derivative of an inverse function.</li> <li>6. Interpret the derivative as a rate of change in varied applied contexts, including velocity, speed, and acceleration.</li> <li>7. Use geometric interpretation of differential equations via slope fields and the relationship between slope fields and solution curves for differential equations.</li> <li>8. Use Euler's method to find a numerical solution of differential equations.</li> <li>9. Use L'Hospital's Rule, including its use in determining limits and convergence of improper integrals and series.</li> </ol>

<b>II. Derivatives (continued)</b>	
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(F) Computation of Derivatives	<ol style="list-style-type: none"> <li>1. Demonstrate knowledge of derivatives of basic functions, including power, exponential, logarithmic, trigonometric, and inverse trigonometric functions.</li> <li>2. Use derivative rules for sums, products, and quotients of functions.</li> <li>3. Use the chain rule and implicit differentiation.</li> <li>4. Calculate the derivatives of parametric, polar, and vector functions.</li> </ol>

<b>III. Integrals</b>	
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(A) Interpretations and Properties of Definite Integrals	<ol style="list-style-type: none"> <li>1. Evaluate a definite integral as a limit of Riemann sums.</li> <li>2. Use basic properties of definite integrals (examples include additivity and linearity).</li> </ol>
(B) Applications of the Integral	<ol style="list-style-type: none"> <li>1. Adapt knowledge of integrals to solve other similar application problems (applications should include finding the area of a region, the volume of a solid with known cross sections, the average value of a function, the distance traveled by a particle along a line, the length of a curve, as well as a curve given in parametric form, and accumulated rate of change).</li> </ol>
(C) Fundamental Theorem of Calculus	<ol style="list-style-type: none"> <li>1. Use the Fundamental Theorem of Calculus to evaluate definite integrals.</li> <li>2. Use the Fundamental Theorem of Calculus to represent a particular antiderivative, and the analytical and graphical analysis of functions so defined.</li> </ol>
(D) Techniques of Antidifferentiation	<ol style="list-style-type: none"> <li>1. Solve antiderivatives following directly from derivatives of basic functions.</li> <li>2. Solve antiderivatives by substitution of variables (including change of limits for definite integrals), parts, and simple partial fractions.</li> <li>3. Evaluate improper integrals.</li> </ol>

<b>III. Integrals (continued)</b>	
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(E) Applications of Antidifferentiation	<ol style="list-style-type: none"> <li>1. Find specific antiderivatives using initial conditions, including applications to motion along a line.</li> <li>2. Solve separable differential equations and use them in modeling (including the study of the equation <math>y' = ky</math> and exponential growth).</li> <li>3. Solve logistic differential equations and use them in modeling.</li> </ol>
(F) Numerical Approximations to Definite Integrals	<ol style="list-style-type: none"> <li>1. Use Riemann sums (left, right, and midpoint evaluation points) and trapezoidal sums to approximate definite integrals of functions represented algebraically, graphically, and by tables of values.</li> </ol>

<b>IV. Polynomial Approximations and Series</b>	
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(A) Concept of Series	<ol style="list-style-type: none"> <li>1. Define a series.</li> <li>2. Define a convergence.</li> <li>3. Explore convergence and divergence through technology.</li> </ol>
(B) Series of Constants	<ol style="list-style-type: none"> <li>1. Explore motivating examples, including decimal expansion.</li> <li>2. Explore geometric series with applications.</li> <li>3. Work with the harmonic series.</li> <li>4. Explain the area of rectangles and their relationship to improper integrals, including the integral test and its use in testing the convergence of a p-series.</li> <li>5. Use the ratio test for convergence and divergence.</li> <li>6. Compare series to test for convergence and divergence.</li> </ol>

IV. Polynomial Approximations and Series (continued)	
CONTENT/KEY CONCEPTS	OBJECTIVES/STANDARDS
(C) Taylor Series	<ol style="list-style-type: none"> <li>1. Explain polynomial approximation with graphical demonstration of convergence.</li> <li>2. Explore Maclaurin series and the general Taylor series centered at <math>x = a</math>.</li> <li>3. Explore Maclaurin series for the functions <math>e^x</math>, <math>\sin x</math>, <math>\cos x</math>, and <math>\frac{1}{1-x}</math>.</li> <li>4. Formal manipulation of Taylor series and shortcuts to computing Taylor series, including substitution, differentiation, antidifferentiation, and the formation of new series from known series.</li> <li>5. Work with functions defined by power series.</li> <li>6. Work with radius and interval convergence of power series.</li> <li>7. Calculate Lagrange error bound for Taylor polynomials.</li> </ol>

<b>V. Technology</b>	
<b>CONTENT/KEY CONCEPTS</b>	<b>OBJECTIVES/STANDARDS</b>
(A) Graphing Calculators	<ol style="list-style-type: none"> <li>1. Use the zoom feature to reveal local linearity.</li> <li>2. Construct a table of values to conjecture a limit.</li> <li>3. Develop a visual representation of Riemann sums approaching a definite integral.</li> <li>4. Graph Taylor polynomials to understand intervals of convergence for Taylor series.</li> <li>5. Use the calculator to draw a slope field.</li> </ol>