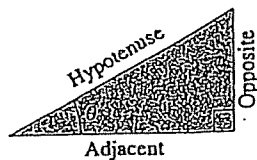


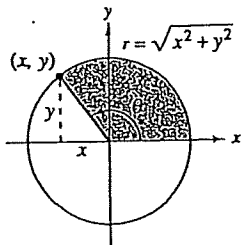
Definition of the Six Trigonometric Functions

Right triangle definitions, where $0 < \theta < \pi/2$.



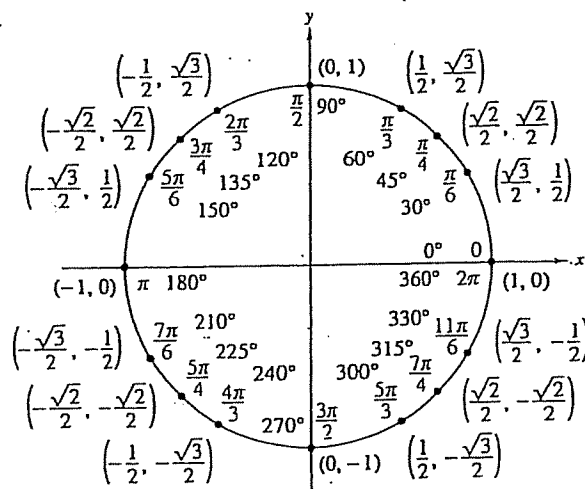
$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}} \end{aligned}$$

Circular function definitions, where θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$

(C o S S i n)



Reciprocal Identities

$$\begin{aligned} \sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x} \end{aligned}$$

Tangent and Cotangent Identities

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

Cofunction Identities

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

Reduction Formulas

$$\begin{aligned} \sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x \end{aligned}$$

Sum and Difference Formulas

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$

Double-Angle Formulas

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

Power-Reducing Formulas

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

Sum-to-Product Formulas

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

Product-to-Sum Formulas

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$

Basic Differentiation Rules

- | | | |
|--|--|--|
| 1. $\frac{d}{dx}[cu] = cu'$ | 2. $\frac{d}{dx}[u \pm v] = u' \pm v'$ | 3. $\frac{d}{dx}[uv] = uv' + vu'$ |
| 4. $\frac{d}{dx}\left[\frac{u}{v}\right] = \frac{vu' - uv'}{v^2}$ | 5. $\frac{d}{dx}[c] = 0$ | 6. $\frac{d}{dx}[u^n] = nu^{n-1}u'$ |
| 7. $\frac{d}{dx}[x] = 1$ | 8. $\frac{d}{dx}[u] = \frac{u}{ u }(u'), \quad u \neq 0$ | 9. $\frac{d}{dx}[\ln u] = \frac{u'}{u}$ |
| 10. $\frac{d}{dx}[e^u] = e^u u'$ | 11. $\frac{d}{dx}[\log_a u] = \frac{u'}{(\ln a)u}$ | 12. $\frac{d}{dx}[a^u] = (\ln a)a^u u'$ |
| 13. $\frac{d}{dx}[\sin u] = (\cos u)u'$ | 14. $\frac{d}{dx}[\cos u] = -(\sin u)u'$ | 15. $\frac{d}{dx}[\tan u] = (\sec^2 u)u'$ |
| 16. $\frac{d}{dx}[\cot u] = -(\csc^2 u)u'$ | 17. $\frac{d}{dx}[\sec u] = (\sec u \tan u)u'$ | 18. $\frac{d}{dx}[\csc u] = -(\csc u \cot u)u'$ |
| 19. $\frac{d}{dx}[\arcsin u] = \frac{u'}{\sqrt{1-u^2}}$ | 20. $\frac{d}{dx}[\arccos u] = \frac{-u'}{\sqrt{1-u^2}}$ | 21. $\frac{d}{dx}[\arctan u] = \frac{u'}{1+u^2}$ |
| 22. $\frac{d}{dx}[\operatorname{arccot} u] = \frac{-u'}{1+u^2}$ | 23. $\frac{d}{dx}[\operatorname{arcsec} u] = \frac{u'}{ u \sqrt{u^2-1}}$ | 24. $\frac{d}{dx}[\operatorname{arccsc} u] = \frac{-u'}{ u \sqrt{u^2-1}}$ |
| 25. $\frac{d}{dx}[\sinh u] = (\cosh u)u'$ | 26. $\frac{d}{dx}[\cosh u] = (\sinh u)u'$ | 27. $\frac{d}{dx}[\tanh u] = (\operatorname{sech}^2 u)u'$ |
| 28. $\frac{d}{dx}[\operatorname{coth} u] = -(\operatorname{csch}^2 u)u'$ | 29. $\frac{d}{dx}[\operatorname{sech} u] = -(\operatorname{sech} u \tanh u)u'$ | 30. $\frac{d}{dx}[\operatorname{csch} u] = -(\operatorname{csch} u \operatorname{coth} u)u'$ |
| 31. $\frac{d}{dx}[\sinh^{-1} u] = \frac{u'}{\sqrt{u^2+1}}$ | 32. $\frac{d}{dx}[\cosh^{-1} u] = \frac{u'}{\sqrt{u^2-1}}$ | 33. $\frac{d}{dx}[\tanh^{-1} u] = \frac{u'}{1-u^2}$ |
| 34. $\frac{d}{dx}[\operatorname{coth}^{-1} u] = \frac{u'}{1-u^2}$ | 35. $\frac{d}{dx}[\operatorname{sech}^{-1} u] = \frac{-u'}{u\sqrt{1-u^2}}$ | 36. $\frac{d}{dx}[\operatorname{csch}^{-1} u] = \frac{-u'}{ u \sqrt{1+u^2}}$ |

Basic Integration Formulas

- | | |
|---|---|
| 1. $\int kf(u) du = k \int f(u) du$ | 2. $\int [f(u) \pm g(u)] du = \int f(u) du \pm \int g(u) du$ |
| 3. $\int du = u + C$ | 4. $\int a^u du = \left(\frac{1}{\ln a}\right)a^u + C$ |
| 5. $\int e^u du = e^u + C$ | 6. $\int \sin u du = -\cos u + C$ |
| 7. $\int \cos u du = \sin u + C$ | 8. $\int \tan u du = -\ln \cos u + C$ |
| 9. $\int \cot u du = \ln \sin u + C$ | 10. $\int \sec u du = \ln \sec u + \tan u + C$ |
| 11. $\int \csc u du = -\ln \csc u + \cot u + C$ | 12. $\int \sec^2 u du = \tan u + C$ |
| 13. $\int \csc^2 u du = -\cot u + C$ | 14. $\int \sec u \tan u du = \sec u + C$ |
| 15. $\int \csc u \cot u du = -\csc u + C$ | 16. $\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$ |
| 17. $\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$ | 18. $\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} \operatorname{arcsec} \frac{ u }{a} + C$ |

GENERAL POWER RULE: $\int u^n du = \frac{u^{n+1}}{n+1} + C, \quad n \neq -1$